Price Twenty-Five Cents.

VOL. I. NO. III.

THE

MATHEMATICAL MONTHLY.

DECEMBER, 1858.

EDITED BY

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CAMBRIDGE:
PUBLISHED BY JOHN BARTLETT.
LONDON:
TRÜBNER AND CO.
1858.

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TERMS.

A single Copy,	$3.00 per annum.
Two Copies to one Address,	5.00 "
Five Copies " "	11.00 "
Ten Copies " "	20.00 "

Payable invariably in advance.

JOHN BARTLETT, *Publisher.*

Entered according to Act of Congress in the year 1858, by
J. D. RUNKLE,
In the Clerk's Office of the District Court of the District of Massachusetts.

CAMBRIDGE:
ALLEN AND FARNHAM, ELECTROTYPERS AND PRINTERS.

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MATHEMATICAL MONTHLY.

Vol. I....DECEMBER, 1858....No. III.

PRIZE PROBLEMS FOR STUDENTS.

I.

In every triangle, the perpendiculars dropped from the angles to the opposite sides meet in the same point; the perpendiculars erected to the middle of its sides meet in the same point; the lines drawn from the angles to the middle of the opposite sides meet in the same point. Prove that these three points are in the same straight line, and that their distances apart are in a constant ratio.

II.

Prove that $\tan 9^0 = \sqrt{5} + 1 - \sqrt{5 + 2\sqrt{5}}$.

III.

Find the coefficient of x^n in the expansion of $\frac{ax + b}{(x-1)(x+2)(x-3)}$ into a series of ascending powers of x.

IV.

The equations $y = ax + a + 1$, $y = (a + 1)x + b$, and $y = bx + 5$, are equations of straight lines in which a and b are positive integers, and the three lines all meet in one point. What values can a and b have, and what are the corresponding coördinates of the common point of the lines?

V.

When the angle between the coördinate axes is $\frac{\pi}{3}$, prove that $x + y - a = \sqrt{xy}$ is the equation of a circle of which the radius is $\frac{1}{3} a \sqrt{3}$.

The solutions of these problems must be received by February 1st, 1859.

ARABIC NOTATION IN MENTAL ARITHMETIC.

BY W. W. NEWMAN.

In all written or practical arithmetics the Arabic notation is introduced and used with the first lessons, because the numbers are so large that the operations would become exceedingly difficult, if pupils were confined to the English printed or written words which name the numbers.

But in mental or intellectual arithmetics there is a diversity of practice, and, of course, a difference of opinion among our most popular authors. Colburn, the celebrated pioneer in this class of works, first uses the figures from 1 to 10 on the fiftieth page, after going through with the simple rules and an introduction to fractions. He also explains the Arabic notation of the numbers from 10 to 100 on the sixty-ninth and seventieth pages; and, with remarkable coincidence, Adams, Perkins, and Thomson do the same at precisely the same place.

On the contrary, Davies, in his New Primary, Greenleaf, Robinson, Stoddard, Emerson, &c., introduce the pupil immediately to the language and practice of the Arabic notation.

Colburn says figures "are not used in the first part of the book, because the pupil would not understand them so well as he will the words," and this is probably the idea of other authors.

But it should be remembered that teachers of the simplest reading lessons find it necessary to teach their pupils the Arabic notation that numbers the pages of their books, and, therefore, the arithmetical language of so small numbers is generally learned before they are introduced to even the simplest primary arithmetic. Again, all the numbers below ten, and all units in larger numbers are presented to the eye by the Arabic notation with a single character; but the shortest of the words has three, and the longest five letters. The tens figures, using only one *figure*, require from four to seven *letters* each. If, therefore, a child cannot understand the figures 8, 16, 98, as well as the English words *eight*, *sixteen*, *ninety-eight*, when they are read alike, and are only two forms of expressing the same things, it must be that a brief, simple mode of spelling only befogs the juvenile intellect, and that silent, unnecessary letters are aids that cannot be dispensed with in primary instruction. Figures are more easily read than words; they are more rapidly written upon the blackboard or slate; they give a condensed and expressive view of operations; they are great improvements upon all previous modes of expressing even small numbers; and the pupil may, without any hinderance, delay, or injury, be introduced to his arithmetical alphabet of ten Arabic characters or letters as his first lesson in the science of numbers. If there are any good and sufficient reasons why these characters should be deferred to the advanced portions of mental arithmetic, we have yet to learn what they are.

DIVISION OF FRACTIONS.

BY A. SCHUYLER.

This subject is usually presented in three cases: — I. To divide a fraction by a whole number. II. To divide a whole number by a fraction. III. To divide a fraction by a fraction.

Though the subject may properly be treated under the above heads, yet it should be understood, that the last case is general, including the other two; for, any whole number can be expressed in the form of a fraction having any denominator. Thus, in general, $n = \frac{n}{1} = \frac{n\,d}{d}$. We shall therefore treat of the general case, III., by reducing the operation to a division of whole numbers.

PRINCIPLES. 1. *If any simple fraction be multiplied by any multiple of its denominator, the result is a whole number.*

Thus, in general, let $\frac{n}{d}$ be any simple fraction, and m any whole number, then $m\,d$ will represent any multiple of the denominator. Then, $\frac{n}{d} \times m\,d = m\,n$, a whole number, since the product of two whole numbers is a whole number.

2. *If both dividend and divisor be multiplied by the same number, the quotient will remain unchanged.* Thus, in general, let D denote the dividend, d the divisor, then $D \div d = D \times m \div d \times m$.

Hence, *If both dividend and divisor be multiplied by any common multiple of their denominators, they will become whole numbers, the quotient will remain unchanged, and will be expressed by a fraction, of which the new dividend is the numerator, and the new divisor the denominator.*

To make the operation as concise as possible, multiply both dividend and divisor by the least common multiple of their denominators.

Let it be required to divide $\frac{3}{4}$ by $\frac{5}{6}$.

OPERATION.

$\frac{3}{4} \div \frac{5}{6} = 9 \div 10 = \frac{9}{10}$

By multiplying both dividend and divisor by 12, the least common multiple of their denominators, 4 and 6, we have $9 \div 10 = \frac{9}{10}$.

Complex and compound fractions should be reduced to simple fractions before division, but mixed numbers need not be thus reduced. Thus, $5\frac{1}{2} \div 2\frac{1}{4} = 22 \div 9 = 2\frac{4}{9}$.

In multiplying both dividend and divisor by the least common multiple of their denominators, cancel as much as possible; and even this should be done mentally, and we should, at once, say

$\frac{3}{4} \times 12 = 9$. In general, $\frac{n}{d} \div \frac{n'}{d'} = n\,d' \div n'\,d = \frac{n\,d'}{n'\,d} = \frac{n}{d} \times \frac{d'}{n'}$, which demonstrates the rule, *Invert the divisor, and proceed as in multiplication.*

It is a serious objection to this rule, that its tendency is to lead the scholar to rely on a mere mechanical process, in which case he soon loses sight of the principle. But the above method keeps the principles constantly before the mind. Let these principles first be thoroughly understood, and the above method will be found very concise and beautiful.

NOTES, BY PROF. ELIAS LOOMIS.

I. My own experience has led me to a very different conclusion respecting decimal fractions from that stated by Mr. Hill, page 3. Instead of "keeping sedulously out of sight the analogy between decimal and vulgar fractions," I would insist that the pupil should constantly remember, not the *analogy* between decimal and vulgar fractions, but the essential *identity* of vulgar fractions with decimals and duodecimals; and for this purpose I would recommend that the pupil be always required to write decimal fractions and duodecimals with a denominator, until he becomes perfectly familiar with the idea that the omission of the denominator is simply a matter of convenience, and only to be allowed in cases where its omission leaves no uncertainty as to what the denominator should be.

II. Mr. Safford, on page 10, has not proved that one of Napier's rules is superfluous, but simply that one rule may be deduced from the other. Napier's rules never professed to inform us of principles not known before, but were designed to embody in a form easily remembered and easily applied, the theorems for the solution of every case in right angled spherical triangles. In this point of view neither of Napier's rules is superfluous, and the two rules together

probably form the most fortunate application of the principles of artificial memory, which the history of the mathematics can furnish.*

ON THE STUDY OF GEOMETRY.

BY PROF. M. C. STEVENS.

THE sentiments expressed by SAMUEL P. BATES, in the first number of the Monthly on the study of Geometry, I consider worthy of the attention of every teacher. Let teachers try the experiment of giving to their classes in geometry, after they have gone over the text-book, some simple problem or theorem not found in the book, and I venture the prediction of a failure in nine cases out of ten, — provided they have not been accustomed to rely on their own powers. If this be so, what does it prove? Most assuredly, that there has been some error in the mode of teaching.

It is my opinion that *pure geometry* receives too little attention in our schools generally; and that the analytical method of arriving at geometrical truths is taking too prominent a place. I think I have a just sense of the importance of analysis and of its power; but I want *pure geometry* to occupy the place it deserves as a means of mental discipline, as well as for its beauty and brevity in the solution of problems. But it is urged that the analytical method is more powerful, because we are led directly to our result; whereas it frequently takes much time to discover the hidden relations that exist in a problem, which must be known in order to apply the other method. All this I am ready to grant, but I urge that greater facility may be acquired in discovering these hidden relations than

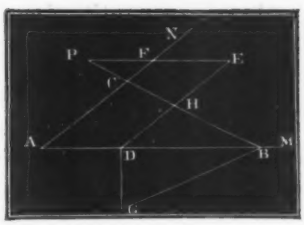
* We suppose that Mr. SAFFORD only intended to show that Napier's rules are not analytically independent; and not that Rule I. should be discarded in practice. If so, then there is no disagreement between Mr. SAFFORD and Prof. LOOMIS. — ED.

is generally supposed. Hence I say, let geometry be taxed to the extent of its power, and not thrown aside too soon.

To this end I think every instructor in mathematics should propose a great variety of geometrical problems to his class for construction. Of course the problems should be selected in reference to the capacity of the class, at first selecting those that are simple, and letting them increase in difficulty by degrees, as the class is able to master them. I have intentionally used the term analysis and geometry in a limited sense. And to show what I really mean by the term geometrical construction, I here introduce the solution to the following problem.

Through a given point without a given angle to draw a straight line, so as to cut off a given area.

CONSTRUCTION.



Let MAN be the given angle, and P the given point. Draw PE parallel to AM, and make the parallelogram $ADEF$ contain the given area. Draw DG perpendicular to AM, and equal to PF. Make $GB = PE$, and join PB, and ABC is the triangle required. Demonstration. The triangles PHE, PCF, and BDH are evidently similar; and by construction $PE^2 - PF^2 = DB^2$. Hence $PHE - PCF$, or $HECF = BDH$. Therefore $ABC = ADEF$. *Q. E. D.*

The reader may compare this with the solution of the same problem as given in Church's Analytical Geometry, and draw his own conclusions. The inference is not to be drawn, however, that the solution there given is not a good one of its kind, and well calculated to illustrate what is intended.

ON THE GENERAL PROPERTIES OF EQUATIONS.

BY L. W. MEECH.

At a certain stage of progress, the student of Algebra, writing down an equation of the third or fourth degree with numeric coefficients taken entirely at pleasure, would suppose there must be some value of x which satisfies the equality. But he would have no adequate idea of the extent to which such coefficients in relation to the roots of the equation are subject to uniform laws, and completely pervaded by them in every part. It was by bringing all the terms to one side so as to make them equal to zero, that Harriot, the companion of Sir Walter Raleigh, in Virginia, first generalized what Cardan and Vieta had well commenced. He showed that the unknown quantity in an equation always has as many values or roots as the index of its power in the first term denotes; he likewise unfolded the composition of the coefficients, and proved them to be represented by the roots taken in regular combinations. The perception of these and other relations has tended to free the science from the amusing enigmas and circumlocutions of the older writers; and widening its sphere has given perspicuity of thought and style to its investigations.

For a practical exercise in these principles, let it be proposed to demonstrate the following new indication of imaginary roots; the given equation being expressed under the general form, with real coefficients,

$$A x^n + B x^{n-1} + C x^{n-2} \ldots + C' x^2 + B' x + A' = 0.$$

Whenever $(n-1)\,B^2 - 2n\,A\,C$ *is negative, or when* $(n-1)\,B'^2 - 2n\,A'\,C'$ *is negative, either or both, there are two or more imaginary roots.*

The proof of this relation is here subjoined in its leading features; a constant reference being presupposed to the latter chapters of any of the works on Algebra which treat of the General Proper-

ties of Equations. Dividing the preceding equation through by A, it is plain, if the roots are all real, the sum of their squares must be positive; since the square root of a negative quantity is imaginary. This sum of the squares is shown in Algebra to be represented by $\frac{B^2}{A^2} - \frac{2C}{A}$; and if this sum be negative, the circumstance must arise not from real, but from imaginary roots, or from a preponderance of the latter. Now to draw this restriction more closely, let us take away the second term of the given equation, by assuming $x = y - \frac{B}{nA}$; in this case the sum of the squares of the values of y will become $\frac{(n-1)B^2 - 2nAC}{nA}$, by proceeding as before. If this also be negative, imaginary roots are the cause; and the denominator may be removed, which does not alter the sign. Lastly, substituting $\frac{1}{z}$ for x, and clearing the given equation of fractions, the preceding reasoning on the first three terms is likewise applicable to the last three terms, and the proposition is demonstrated.

When the coefficients are real, "imaginary roots always enter in pairs," and if n is made equal to 2 in the preceding criteria, we have essentially the same test of imaginary roots, which is presented by the common solution of the quadratic equation. It should further be remarked, that there is no simple and universal criterion of imaginary roots; the preceding test, as well as Descartes' rule of signs, admits of occasional application.

NOTE ON THE LEVER.

BY R. G. HATFIELD.

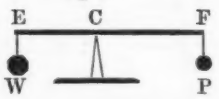
That the weight and the power, when in equilibrio, are in proportion, inversely, as their respective distances from the fulcrum, may be demonstrated as follows: —

A rod of uniform section, and of some homogeneous material

resting at its centre on a point of support, as at C, Fig. 1, may be supposed to be divided at some point D, into two unequal lengths, and the weight of these two unequal portions may be supposed concentrated at their respective centres of gravity E and F, without destroying the equilibrium. This is represented in Fig. 2. The line EF being supposed rigid and without weight: W, being equal to the weight of the part of the rod AD, and P, to the part DB.

Fig. 1.

Fig. 2.



Thus we have the power P, acting upon the weight W, through the lever EF, and the power and weight are in a state of equilibrium. Now it is to be shown that $W:P::CF:EC$; or, since W equals the weight of AD and P, the weight of DB, and since, as the rod is uniform throughout its length, the lengths of the parts are in proportion as their weights, and may be taken to represent their weights respectively, therefore it will be sufficient to show that $AD:DB::CF:EC$. Since ED is the half of AD, and DF the half of DB, therefore EF is the half of AB; hence EF is equal to AC or to CB.

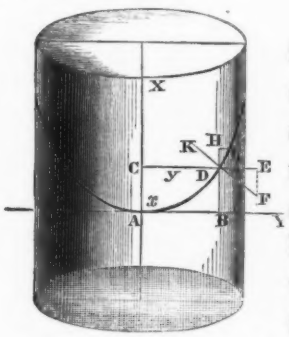
First, $EF = AC$: and these are each equal the sum of their parts, or, $EC + CF = AE + EC$. Cancelling the identical terms, there remains $CF = AE$; and since $AE = \frac{AD}{2}$, therefore, $CF = \frac{AD}{2}$.

Second, $EF = CB$ as above shown; and these are each equal the sum of their parts, or, $EC + CF = CF + FB$. Cancelling the identical terms, there remains $EC = FB$; and since $FB = \frac{DB}{2}$, therefore, $EC = \frac{DB}{2}$. Now since EC equals the *half* of DB, and CF equals the *half* of AD (as above), therefore is $EC:DB::CF:AD$, and by transposing the extremes, $AD:DB::CF:EC$.

A FLUID PARABOLIC MIRROR.

BY PROF. GEORGE R. PERKINS.

If an open vertical cylinder, containing a fluid, is made to revolve with a uniform motion about its axis, the upper surface of the fluid will assume a perfect parabolic form.



If we consider a particle of the fluid at D, it must, in reference to the surrounding particles, remain in equilibrium. Hence, the resultant of the forces acting upon this particle must be in the direction of the normal to the surface assumed by the fluid, that is, it must be in the direction of DF. Consequently the force of gravity must be to the centrifugal force as EF is to ED. If we call the coördinates of this point x and y, we shall have in the differential triangle DHG, $HD = dx$; $HG = dy$; but the triangle DHG is similar to the triangle DEF, hence, $dy : dx ::$ force of gravity : centrifugal force. Denoting, as usual, the force of gravity by g, and by T the time in seconds of one revolution of the cylinder, we shall have $\frac{2\pi y}{T}$ for the velocity, of the point at D. But the centrifugal force is expressed by the square of the velocity divided by the radius, that is, we have $\left(\frac{2\pi y}{T}\right)^2 \div y = \frac{4\pi^2 y}{T^2}$, for the centrifugal force. Hence, the above proportion becomes

$$dy : dx :: g : \frac{4\pi^2 y}{T^2} \text{ or,}$$

$$\frac{4\pi^2 y\, dy}{T^2} = g\, dx, \text{ that is, } 2y\, dy = \frac{g\, T^2\, dx}{2\pi^2}.$$

Integrating, we have

$$y^2 = \frac{g\, T^2 x}{2\pi^2}.$$

This is the equation of a Parabola having $\frac{g\, T^2}{8\pi^2}$ for its focal dis-

tance. If we denote by N the number of revolutions in each minute, we shall have $N = \frac{60}{T}$.

Calling $g = 32.1908$ feet; $\pi = 3.14159$, we shall have the focal distance in feet expressed by $T^2 \times 0.4073 \ldots = \frac{1466.33 \ldots}{N^2}$. From this expression we see that the focal distance varies inversely as the square of the angular velocity.

We deduce from the above expression the following numerical results: —

No.	Focal distance in feet.	No.	Focal dist. in feet.
1	1466.33	20	3.66
2	366.58	30	1.63
3	162.92	40	0.92
4	91.64	50	0.59
5	58.65	60	0.41
6	40.73	70	0.30
7	29.92	80	0.23
8	22.91	90	0.18
9	18.10	100	0.15
10	14.66	120	0.10

From these results we see that when the fluid revolves sixty times in one minute, that is, once in each second of time, the focal distance will be 0.41 feet = 4.92 inches. If it revolve once in six seconds, the focal distance will be 14⅔ feet nearly. If it revolve once in ten seconds, the focal distance will be 40¾ feet, nearly.

Is it possible to make use of this kind of mirror for Astronomical purposes?

In theory, this fluid mirror will be perfect in its parabolic form, whatever the velocity of revolution may be, provided it is uniform. Each and every ray of light falling upon it parallel to its axis will be reflected to the same focal point. Whatever change the fluid may undergo by reason of a change in the atmospherical temperature cannot alter the parabolic form of the surface, since this form does not depend at all upon the specific gravity of the fluid, but

only upon the velocity of rotation, consequently this mirror will remain perfect at all temperatures, which must give it great advantages over the ordinary solid mirrors.

And since the focal distance does not depend upon the size of the mirror, it follows, theoretically, that there is no limit to the size of mirrors constructed in this way, consequently any amount of light may be used which will allow of high magnifying powers in the eye-glasses, and in this consists the great telescopic power which we shall in this way be able to obtain.

As yet, we have given no method by which objects out of the zenith can be viewed.

This must be accomplished by first receiving the rays of light upon a plane mirror, so adjusted as to throw them after reflection vertically upon the parabolic mirror. This plane mirror will of necessity absorb more or less of the direct rays, still as the size of the mirror is unlimited, I suppose no serious objection would result from this. I apprehend the most difficult thing to be overcome in this case is to construct a plane mirror of such vast size which shall be sufficiently accurate to answer our purpose. For this plane mirror must be solid, and subject to all the defects incident to mirrors so formed.

But suppose this fluid parabolic mirror can only be employed in a zenith telescope, does it not promise advantages sufficiently great to warrant the expense of all necessary experiments to test its practicability?

By using mercury, which possesses the speculum surface but little short of perfection, and which also is of such specific gravity as to become very stable after having once assumed its position of equilibrium, it would seem that in the present state of mechanical perfection we may reasonably hope to succeed in constructing in this way reflecting telescopes of almost unlimited power.

Should this plan succeed, I need not point out its great importance in a scientific point of view. The astronomer will then have the ability of deciding many questions in regard to the theory of nebulous bodies, to say nothing about the distinctness with which it must of necessity expose to view the surface of the moon. The question so often asked, as to whether the moon is inhabited, may then be at once answered without a possibility of error.

Some of the most perfect mirrors which it is possible for us to conceive of, are formed by the physical laws of nature, in the atmosphere, in the case of *mirage*. This fact is well calculated to encourage the hope of success in the case of mercurial parabolic mirrors.

DEMONSTRATION OF PROF. PEIRCE'S PROPOSITIONS ON THE DISTRIBUTION OF POINTS ON A LINE.

BY MAJOR R. J. ADCOCK.

PROPOSITION (1). Let the number of points upon a line whose length is a be represented by a, and the number upon any part x measured off from one end by x. If the line x move along the line a until its forward extremity reach the other end of a the two points at the extremities of x will evidently have taken $(a - x)$ positions upon a. Hence the whole number of possible positions of two points upon a, which are never at a greater distance than x, equals the sum of all the remainders $(a - x)$ obtained by varying x from 0 to x. This sum $= \int_0^x (a - x)\, dx = ax - \frac{1}{2}x^2$. The whole number of possible relative positions of two points upon a is $\int_0^a (a - x)\, dx = \frac{1}{2}a^2$. When $ax - \frac{1}{2}x^2 = \frac{1}{2}$ of $\frac{1}{2}a^2$, $x = a(1 \pm \frac{1}{2}\sqrt{2}) = 0.29289 \times a$.

PROP. (2). When $x = \frac{1}{2}a$, $ax - \frac{1}{2}x^2 = \frac{3}{8}a^2 = \frac{3}{4} \times \frac{1}{2}a^2$.

PROP. (3). When $x = \frac{1}{3}a$, $ax - \frac{1}{2}x^2 = \frac{5}{18}a^2 = \frac{5}{9} \times \frac{1}{2}a^2$.

PROP. (4.) When $x = \frac{2}{3}a$, $ax - \frac{1}{2}x^2 = \frac{8}{18}a^2 = \frac{8}{9} \times \frac{1}{2}a^2$.

PROP. (5). Since, when the line returns into itself, the greatest distance of two points can be only a, two points at a distance x, can only take a positions; the sum of which from 0 to $x = \int_0^x a\,dx = ax$.

PROP. (6). The number of positions of two points upon x is $\frac{1}{2}x^2$.

The number of these, with another point on dx, is $\frac{1}{2}x^2\,dx$ positions of three points. Hence the whole number of positions of three points upon x is $\int_0^x \frac{1}{2}x^2\,dx = \frac{1}{6}x^3$. When the line returns into itself, the longest possible distance of two points may still be regarded as a, and the whole number of positions of three points upon it, is $\frac{1}{6}a^3$.

Let x be the least distance of the two nearest points, then $2x$ is the least distance that can be occupied by three points. Beginning with this position, let the two forward points, remaining at the same distance, move around the curve until the advance point is at the distance x from the fixed point. The three points will thus have taken $(a - 3x)$ positions. Hence $\int_0^x (a - 3x)\,dx$ is the number of positions of three points when one is fixed, and the other two never at a greater distance than x, nor than either of their distances from the fixed point. But the fixed point can be moved a distance a from its first position. Hence the whole number of positions of three points under the conditions is $a\int(a - 3x)\,dx = a^2x - \frac{3}{2}ax^2 + C$. Taking the integral between the limits 0 and x, we have $a^2x - \frac{3}{2}ax^2$ for the number of positions when the two nearest points are at a less distance than x. When $a^2x - \frac{3}{2}ax^2 = \frac{15}{16} \times \frac{1}{6}a^3$, $x = (\frac{1}{3} \pm \frac{1}{12})\,a = \frac{5}{12}a$, or $\frac{1}{4}a$.

PROP. (7). The constant of integration in $a^2x - \frac{3}{2}ax^2 + C$, is $\frac{1}{6}a^3$, for when the least distance of the two nearest points is 0, the whole number of positions is $\frac{1}{6}a^3$. When $a^2x - \frac{3}{2}ax^2 + \frac{1}{6}a^3 = \frac{1}{2} \times \frac{1}{6}a^3$, $x = \left(\frac{1 \pm \frac{1}{2}\sqrt{2}}{3}\right)a = 0.09763\,a$.

PROP. (8). When $a^2x - \frac{3}{2}ax^2 = \frac{1}{6}a^3$, $x = (\frac{1}{3} \pm 0)\,a$.

When $a^2x - \frac{3}{2}ax^2 = \frac{3}{4} \times \frac{1}{6}a^3$, $x = (\frac{1}{3} \pm \frac{1}{6})\,a = \frac{1}{2}a$ or $\frac{1}{6}a$.

When $a^2x - \frac{3}{2}ax^2 = \frac{6}{6}\frac{3}{4} \times \frac{1}{6}a^3$, $x = (\frac{1}{3} \pm \frac{1}{24})\,a = \frac{3}{8}a$ or $\frac{7}{24}a$.

The three points are evidently always included in an arc equal to a, minus the greatest distance of two points in any case.

Hence the arc which includes the positions $\frac{1}{6}a^3$ is $\frac{2}{3}a$.

Hence the arc which includes the positions $\frac{3}{4} \times \frac{1}{6}a^3$ is $\frac{1}{2}a$.

Hence the arc which includes the positions $\frac{6}{6}\frac{3}{4} \times \frac{1}{6}a^3$ is $\frac{5}{8}a$.

Since by Prop. (5), the number of positions of two points included in an arc x, less than a, is ax, the number of positions of three points included in x is $\int ax\,dx = \frac{1}{2}ax^2$.

These propositions might be demonstrated by the Theory of Permutations and Combinations.

THEORY OF THE DISTRIBUTION OF POINTS ON A LINE.

BY W. P. G. BARTLETT.

LET l be the length of a line returning into itself, and dl the distance between two of its consecutive points. Let it be divided into n intervals by n points. Then obviously the coefficient of x^l in the development of

$$(1) \qquad [x^0 + x^{dl} + x^{2dl} + x^{3dl} \ldots\ldots + x^{\frac{l}{dl}dl}]^n,$$

will express $n!$* times the number of different sets of n intervals

* Using the notation $n!$ to express the product $1.2.3.4\ldots\ldots n$

into which the line may be divided. Putting $x^{dl} = z$ and $\frac{l}{dl} = p$, (1) becomes

$$(2)\quad [1+z+z^2+z^3+\ldots\ldots+z^p]^n = \left(\frac{1-z^{p+1}}{1-z}\right)^n = (1-z^p)^n (1-z)^{-n}$$
$$= (1 - A_1 z^p + A_2 z^{2p} - A_3 z^{3p} + \ldots\ldots \pm A_n z^{np})$$
$$(1 - B_1 z + B_2 z^2 - B_3 z^3 + \ldots\ldots),$$

in which A and B are the binomial coefficients for positive and negative values of n respectively.* The coefficient of $x^l = z^p$ in (2) is $\pm B_p - A_1 = \pm B_p = \frac{(n+p-1)!}{p!\,(n-1)!} = \frac{p^{n-1}}{(n-1)!}$.

$$(3)\qquad \therefore \frac{p^{n-1}}{n!\,(n-1)!} = \frac{l^{n-1}}{n!\,(n-1)!\,dl^{n-1}}$$

is the number of different sets of n intervals into which the line l can be divided.

To find in how many of these sets the least interval is x, we may suppose the excess of l over nx to be distributed in all possible ways (no reference being had to the order of the parts), among the remaining $n - 1$ intervals, giving therefore

$$(4)\qquad \frac{(l-nx)^{n-2}}{(n-1)!\,(n-2)!\,dl^{n-2}}$$

different sets of n intervals, in each of which the least interval is x. But as the different orders in which each set of intervals may be distributed over the line depend only on their number n, we may divide (4) by (3) and obtain

$$(5)\qquad \frac{n(n-1)(l-nx)^{n-2}dl}{l^{n-1}},$$

as the *probability* of the least one of the intervals being x. Putting dx for dl, we may integrate (5) with reference to x between any limits greater than 0 and less than $\frac{l}{n}$, and thus obtain as the *probability* of the least interval being included between a and b,

$$(6)\qquad \int_a^b \frac{n(n-1)(l-nx)^{n-2}}{l^{n-1}}dx = \frac{(l-na)^{n-1} - (l-nb)^{n-1}}{l^{n-1}}.$$

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* This method I derived from the Introduction to BREMIKER's Six Place Logarithms.

When $a = 0$, (6) becomes $\frac{l^{n-1} - (l - nb)^{n-1}}{l^{n-1}}$, which may also be obtained by distributing the excess $l - nb$ in all possible ways (without regard to the order of the parts), among the n intervals, and taking its complement to unity.

To find in how many of the sets (3) all the points are just included in a length y, we may suppose y to be divided in all possible ways (without regard to the order of the parts), into $n - 1$ parts, *provided* that no part shall be greater than $l - y$. Then (2) becomes, putting $\frac{y}{dl} = q$,

$$(7)\quad [1 + z + z^2 + \ldots\ldots + z^{p-q}]^{n-1} = (1 - A'_1 z^{p-q} + A'_2 z^{2(p-q)} - A'_3 z^{3(p-q)} + \ldots\ldots \pm A'^{(n-1)(p-q)}_{n-1})(1 - B'_1 z + B'_2 z^2 - B'_3 z^3 \ldots\ldots),$$

in which A' and B' are the binomial coefficients for the $(n-1)$st power, and the coefficient of z^q, divided by $(n-1)!$, is the number of ways in which y is required to be divided. The general form of this coefficient, retaining only the infinite terms, is

$$\pm B'_q \pm A'_1 B'_{2q-p} \pm A'_2 B'_{3q-2p} \pm A'_3 B'_{4q-3p} \pm, \text{ \&c.}$$

$$= \frac{q^{n-2}}{(n-2)!} - (n-1)\frac{(2q-p)^{n-2}}{(n-2)!} + \frac{(n-1)(n-2)}{2!}\cdot\frac{(3q-2p)^{n-2}}{(n-2)!} - \text{ \&c.},$$

in which there can be no more than n terms, and only those are to be retained in which the subscript number to B' is positive. Whence dividing by $(n-1)!$ and by (3) and substituting the values of p and q, we get, since $dy = dl$,

$$(8)\qquad \frac{n(n-1)y^{n-2}}{l^{n-1}}dy - \frac{n(n-1)^2(2y-l)^{n-2}}{l^{n-1}}dy + \frac{n(n-1)^2(n-2)(3y-2l)^{n-2}}{2!\,l^{n-1}}dy - \text{ \&c.},$$

as the *probability* (omitting the terms which become negative), of all the points being included in a length y. If we integrate (8), taking care to include no negative term between the limits, and remembering that y cannot be greater than $\frac{(n-1)l}{n}$, we have

$$(9) \qquad \int_a^b \frac{n(n-1)y^{n-2}}{l^{n-1}}dy - \int_{a'}^b \frac{n(n-1)^2(2y-l)^{n-2}}{l^{n-1}}dy + \int_{a''}^b \frac{n(n-1)^2(n-2)(3y-2l)^{n-2}}{2!\,l^{n-1}}dy - \&c.,$$

in which $b < \frac{(n-1)l}{n}$ and $a' > \frac{l}{2}$, $a'' > \frac{2l}{3}$, &c. The general integral of (9) is

$$(10) \quad \frac{ny^{n-1}}{l^{n-1}} - \frac{n(n-1)(2y-l)^{n-1}}{2!\,l^{n-1}} + \frac{n(n-1)(n-2)(3y-2l)^{n-1}}{3!\,l^{n-1}} - \&c.,$$

which vanishes for any value of n when y is taken at its lowest limits, but when y is taken at its highest limit, $\frac{(n-1)l}{n}$, becomes

$$n\left(\frac{n-1}{n}\right)^{n-1} - \frac{n(n-1)}{2!}\left(\frac{n-2}{n}\right)^{n-1} + \frac{n(n-1)(n-2)}{3!}\left(\frac{n-3}{n}\right)^{n-1} - \&c.,$$

$= 1$, for any value of n.

It is easy to see that on a straight line, not returning into itself, the number of possible positions for n points is

$$(11) \qquad \frac{p^n}{n!} = \frac{l^n}{n!\,dl^n},$$

and the number of possible positions in which the extreme points are x apart is $\frac{l-x}{dl} \times$ (number of possible positions on x for $(n-2)$ points) $= \frac{(l-x)x^{n-2}}{(n-2)!\,dl^{n-1}}$; which divided by (11) gives

$$\frac{n(n-1)(l-x)x^{n-2}dl}{l^n},$$

as the *probability* that all the points are just included in x. This may be integrated between any limits less than l, and thus gives

$$(12) \quad \int_a^b \frac{n(n-1)(l-x)x^{n-2}dx}{l^n} = \frac{n(b^{n-1}-a^{n-1})}{l^{n-1}} - \frac{(n-1)(b^n-a^n)}{l^n},$$

as the *probability* that all the points are included in a length greater than a and less than b.

Formulæ (6), (10), and (12), are sufficient to solve all the cases presented in the October Number of the Monthly. The decimal in Prop. 1, equal to $1-\sqrt{\frac{1}{2}}$, and that in Prop. 7, equal to $\frac{1}{3}(1-\sqrt{\frac{1}{2}})$ may be found by putting (12) and (6) equal to $\frac{1}{2}$, and solving with reference to b, a being put equal to 0.

AN ACCOUNT OF THE COMET OF DONATI. 1858.

PART II.

For the following details, relating to the appearance presented by the comet in the telescope or to the naked eye, use has been made principally of the manuscript records of the Observatory of Harvard College, the results of observations made elsewhere not being accessible through the ordinary channels of information at the time of writing.

There was a marked increase of brilliancy accompanied by an equally perceptible lengthening of the tail, between the 10th and the 25th of September. Its sudden advance in size and splendor during the week following the latter date, was in perfect keeping with the often repeated history of bodies of its class. Every condition seemed to favor a rapid development. It was approaching the sun, which not only subjected it to a more intense illumination, but probably to other influences of a nature not well understood, but perfectly obvious in their effects; it was drawing nearer also to the earth, and was, besides, every night taking a more favorable position above the horizon, and presenting to better advantage the length and curvature of its train; lastly, the absence of moonlight towards the end of the month contributed more than any other circumstance to enhance the grandeur of the scene. Scarcely less impressive was the sudden vanishing of the spectacle a few weeks later, attributable to similar influences as it were reversed; — the comet receding from the sun and earth, contracting its dimensions, descending low in the mists of the horizon, and finally almost extinguished by the returning moonlight.

On the 8th, the diameter of the nucleus was ascertained to be two thousand miles. In immediate contact with it, was an intensely brilliant nebulosity, having a diameter of about three thousand

miles, while the surrounding diffused light extended forty or fifty thousand towards the sun. Measured by ordinary standards, this latter distance appears large, but it was manifestly insignificant compared with the effusion of nebulosity in the direction of the tail. Indeed the comparative absence of any considerable collection of diffuse light on the side nearest the sun, outside of the above radius, was so noticeable as to excite remark on several subsequent occasions. The fact of the position of the nucleus, precisely in the vertex of the train, must have been generally noticed. At this date the tail had acquired a length of sixteen millions of miles.

To ascertain the true dimensions of the nucleus, and to compare the intensity of its light with that of a star of equal brightness, the comet on the morning of the 9th was kept in the field of the great refractor by the clock-work motion, and the effect of the approach of daylight upon it, carefully noted.

In the early twilight the nucleus resembled a star of the fifth magnitude, subtending an angle corresponding to a diameter of five thousand miles; but owing partly to atmospheric disturbances, and partly to the difficulty of distinguishing its precise border, this proved to be much too large, for it diminished to less than half that amount when the daylight had become sufficiently strong to obliterate all but the true centre, which continued in sight until twelve minutes before sunrise; the light however no longer retained the scintillating, star-like character which distinguished it when seen on the background of a dark sky. At this time, therefore, the nucleus must have been nearly of the size of our moon, and probably shone with somewhat inferior intrinsic brightness.

Sept. 12. "A rapid increase in brightness, and length of train, the latter covers an arc of 6°."

"The intensity as well as the quantity of light emanating from the nucleus are the most distinctive features. To the naked eye, aided by the light of the envelope and contiguous part of the tail, it

was as bright as a star of the third magnitude. In the telescope the light concentrated within a circle of 10″ diameter, or six thousand miles, resembles that of a star of the fifth or sixth magnitude diffused over an equal space." The view of the comet on the morning of the 13th was still more satisfactory, owing to its greater elevation and the absence of moonlight.

To the naked eye on the 17th, the head equalled a star of the second magnitude. Its southern side (on the left hand and below as seen in the evening) was decidedly the brightest. A similar contrast was noticeable through a considerable extent of the tail for several weeks following; the convex outline being both brighter and more clearly defined than the opposite side. Ultimately this distinction disappeared, or rather it was reversed; the change taking place gradually, and becoming most noticeable after the 6th of October.

The commencement of a most important epoch in the physical history of the comet, dates in our records from the 20th of September. It is probable that symptoms of approaching changes, faintly indicated, may have appeared somewhat earlier; they were not however noticed on the 17th and 18th, on both of which occasions the comet was observed, though particular attention was not then given to the condition of the nucleus.

On the evening of the 20th, the train at its origin was plainly bifurcated, issuing from the head in two unequal streams forming its two sides, and leaving between them a dark space behind the nucleus. Their outline was a curve resembling an arc of the parabola or hyperbola. The southern stream was so much the more brilliant of the two, that in strong twilight this alone would have been seen as a short tail inclined by 30° or more to the true axis. "Between the nucleus and the sun is interposed an obscure crescent-shaped outline within which the light is unequally distributed, and has a strangely confused chaotic look; the details are too undecided

for precise description. There is also an elongation of the nucleus, which is singularly brilliant, or perhaps a ray extending a few seconds from it on the following or upper side. The exact character of these phenomena could not be made out, but they seemed to indicate the presence of some internal disturbing force."

Fig. 3 * is taken from a sketch made at the time. The exterior dotted line represents the outline of the head of the comet, *n* the nucleus with the position of the ray, and *a a′ a″* the place of the obscure band. The latter was so faintly marked that it might easily have been overlooked, or have passed for an illusion.

Fig. 3.

Sept. 23d. "A fine clear sky with the moon nearly full. To the naked eye the head of the comet is as bright as a star of the first magnitude, and the train, notwithstanding the moonlight, is 6° or 8° long. It is already a brilliant object half an hour after sunset. The telescopic view is most extraordinary. The nucleus has diminished in size, being now only 3″, or 1300 miles, in diameter. Its light is exceedingly intense, and somewhat more concentrated than on the 20th. Outside of it is a bright envelope with its vertex in the direction of the sun, and 15″, or 6400 miles, distant. This is bounded on its outer margin by a dark band. The boundary of a second and less brilliant envelope is distant at its vertex about 30″, or 12800 miles, from the nucleus, and is terminated by a similar dark arch,

* In the figures the head of the comet is inverted, so as to correspond with its appearance in the telescope.

outside of which, again, is an atmosphere of faint, diffused nebulosity rapidly shaded off. The outlines can be distinguished through an arc of 220° or more, reckoned from the nucleus, but they extend considerably further into the train on the following or bright side."

Fig. 4.

The form and situation of the envelopes and their relative degrees of brilliancy are represented in Fig. 4 by gradations in the character and strength of the lines in the engraving.* The scale to which this cut is drawn has not been ascertained from exact micrometer measurements, and is not to be implicitly relied on in comparing the dimensions of the envelopes at this date with others where the proper angles at several points were more carefully determined.

Sept. 24th. The train is 7° in length, and evidently curved as represented in Fig. 5, which gives its appearance as seen at evening in the north-west. The telescopic view showed a decidedly dark axis to the tail, extending close up to the nucleus, which was elongated in a direction perpendicular to the axis. The inner envelope south of the nucleus was in part separated from it by a darker space and was twice as bright as the one

Fig. 5.

* The parallel curved lines in the wood-cut are employed merely to indicate the

next outside of it, which in turn was much brighter than the exterior nebulosity.

It is possible that the envelope, of which the outer edge is indicated in Fig. 6 by the letters a a' a'', corresponds to the bright nebulosity comprised between a a' a'', and n in Fig. 3, and that the outline $b\,b'\,b''$ was formed between the 20th and the 23d. The dimensions of the two envelopes on the 24th, were found to be as follows. The letters refer to Fig. 6.

Fig. 6.

$n\,b' = 5800$ miles, $\quad n\,a' = 13400$ miles,

$b\,b'' = 17000$ " $\quad a\,a'' = 31000$ "

The obscure bands on the margins of the envelopes were 3″. 5 or 1400 miles broad. The estimated angle of divergence of the two forks of the tail at a distance of 10′ from the nucleus was from 20° to 25°.

On the 25th, the nucleus presented itself under a new aspect, in the act, as it afterwards proved, of disengaging a new envelope, or rather in a stage preparatory to that event. This perhaps is the first instance where an envelope has been seen in embryo at the surface of the nucleus, and has been traced through successive stages to a full development. The same phenomenon was subsequently illustrated in the case of the present comet by several exhibitions of

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places where the light was more or less intense, and must not be too literally interpreted. The general effect is better given in Fig. 1; but all attempts at a faithful representation by means of an engraving, must, from the necessity of the case, be inadequate.

a similar nature; their history has a peculiar value, because it affords an insight into the mysterious processes by which the train is thrown out from the nucleus, under the stimulating influence of the sun's light and heat, or possibly of some unknown emanation from the same source. The following were the most conspicuous gradations of light recognized in its neighborhood. Commencing with the dark axis we have: —

1st. The axis, a narrow, well defined dark stripe penetrating quite up to the central body. Next in order towards the sun is the nucleus, on the eve, as we may say, of an eruption. The expression is fully warranted by its subsequent history. When seen to best advantage, two little streams of luminous matter were observed issuing from it one on each side, doubtless on their way to supply material to the tail now so rapidly expanding. Outside of the nucleus and of the nebulosity, apparently adhering to it, was a comparatively dark space, succeeded by the envelopes $b\,b'\,b''$ and $a\,a'\,a''$ with the intervening dark bands, and lastly, over the whole a thin veil of diffuse light; the latter attaining a distance of seventy thousand miles. If we include the dark axis, and the dark background of the sky, we have here nine alternations of light and shade, of various grades of intensity.

Fig. 7.

The micrometric measurements furnish the following dimensions, the letters referring to Fig. 7.

$n\,b' = 7100$ miles,

$b\,b'' = 15600$ "

n to dark space outside of $c' =$ 3500 miles. This space, first seen on the 24th, continued to enlarge until the envelope $b\,b'\,b''$ was completely severed from con-

nection with the nucleus. The breadth of the obscure band outside of $b\,b'\,b''$ was 1600 miles. At a distance of about three hundred thousand miles from its origin, the breadth of the tail was found to be one hundred and forty thousand miles. Its extreme length was 11°, and the breadth where widest 1°.

Sept. 27th. We have now conclusive evidence that the condition of the central luminosity on the 24th and 25th was that of an envelope in its earliest stages. "The outline of a new envelope is clearly distinguished. In form and position it is a miniature of that which has hitherto been the innermost. Like the latter it sets awry, inclining to the right hand side of the axis, as represented in Fig. 8. The outline of $a\,a'\,a''$ of Fig. 7 is becoming indistinct. The narrow dark stripe in the axis having its vertex precisely at the nucleus, is a remarkable object;" its width near its origin was found to be 1800 miles.

Fig. 8.

Fig. 9.

The tail had now attained a length of 13°, or eighteen millions of miles. A new appendage in the form of a long, narrow ray issuing from its convex side was seen as represented in Fig. 9, not following the curve of the tail proper, but projected nearly in a straight line from the sun. Its appearance simultaneously with the throwing off of a new envelope suggests the possibility of the two phenomena being in

some way connected with each other; both, it will be noticed, lie on the same side of the axis. Supposing it to have started from the head of the comet on the 25th, its velocity must have reached eight or ten millions of miles daily. Other comets have exhibited similar rays. That of 1843 shot out its streamers to a much greater distance. One which appeared in 1744 is said to have had no less than six, spread out like a fan.

On the 28th, the image of the nucleus in the focus of the large refractor afforded distinct photographic action, but the surrounding luminosity was not intense enough to form a picture. "The dark opening in the axis of the tail occupies about one twelfth of its breadth at a distance of 1° from the nucleus; it may be traced distinctly one or two degrees. The head of the comet, seen with a small telescope in strong twilight, which obliterates all but this brightest portion, is crescent-shaped as in Fig. 10. The tail is 19° long, or twenty-six millions of miles, with a streamer as on the 27th."

Fig. 10.

Sept. 29th. Between this date and the 30th, the comet passed its point of nearest approach to the sun, being distant about fifty millions of miles, and not quite seventy millions from the earth, which it was still rapidly approaching.

"Marked changes have occurred since the 27th. The little half moon envelope, then closely shrouding the nucleus, has elevated itself above it, and become the most conspicuous feature in the telescopic view. It is brightest near its outer edge," an indication that it was about to separate from the central nebulosity.

Fig. 11 will serve to convey an idea of the disposition of the envelopes; $a\,a'\,a''$ was fast losing its contour, which could with difficulty be made out; its place in the figure is not much to be

trusted. It will be remembered, that, within five days, all the nebulosity within the outline $c\,c'\,c''$ had been thrown off from the surface of the nucleus, rising from it at the rate of a thousand miles daily.

Fig. 11.

There is reason to suppose that the evolution was attended with something of violence, or of the nature of a sudden disruption, or of an explosion, if the expression does not convey too much the idea of motion apparent to the eye. There were rays or jets of light streaming in different directions from the centre, one in particular on the following (apparent right hand) side, imperfectly suggested on the 27th, now plainly seen, and there was a general aspect of confusion, suggesting the idea of internal disturbances. It was afterwards observed that as the nebulous matter rose higher and higher above its origin, it became uniformly blended, as if, when relieved from the immediate neighborhood of the nucleus, it was disposed to an even and symmetrical arrangement. The measured arcs gave the following results, n in Fig. 11 designating as usual the nucleus, and b' and c' the vertices of the envelopes.

$$n\,b' = 10500 \text{ miles},$$
$$n\,c' = 6000 \text{ “}$$

The thickness of the brightest part of the arch under b' was 2000 miles. Since the 24th, b' has ascended (towards the sun) about five thousand miles, or only about one three thousandth part of the distance over which the end of the tail has advanced during the same interval.

Sept. 30th. “The edge of the envelope $c\,c'\,c''$ is very distinct,

and may be traced through an angle of 270°, reckoned from the nucleus. The latter is truncated, as it has often before been seen on the side opposite to the sun, giving it a half moon shape. The dark axis, which at its origin is almost black, and is of even breadth with the nucleus, completes the resemblance to a *phase* and *shadow*." There are objections to this explanation, although at first sight it is very plausible. Each new envelope as it emerges from the nucleus has the same phase-like form, while it is certainly everywhere permeated by the sunlight, a very small envelope still adhering to the nucleus would thus explain the peculiar form of the latter. The dark axis occupies a larger proportion of the whole breadth of the train at a distance of several degrees from the nucleus, than can with any probability be attributed to the defect of light intercepted by so small a body. It is moreover curved, which could not happen to a sensible amount in the shadow.

Perhaps two phenomena are here superimposed; a comparative deficiency of nebulosity towards the central regions of the tail, and an actual shadow perceptible a short distance only, close to the head of the comet, where at any rate we must assume the existence of a considerable collection of nebulous matter, sufficient to exhibit the outlines of a shadow cast upon it, if such really exists. This view receives some confirmation from a note of a later date. "The outlines of the axis-band are *straight lines* near the nucleus, but at a little distance they begin to blend with the general deficiency of light in the middle of the train." It seemed here to be conceivable that the shadow-margin and the outlines of the axis were distinct phenomena.

The tail to the naked eye was 22° long, or twenty-six millions of miles, and from 2° to 3° broad near its extremity, where also its rate of curvature was pretty suddenly increased. The upper outline was throughout brightest and best defined.

Oct. 2d. No new envelope had yet been formed, nor were any

indications of its approach manifested, although they were carefully looked for in the expectation that one would shortly appear. The nucleus, however, was unusually bright, and rounded on the side toward the sun. An increase of brilliancy in the nucleus was afterwards recognized as the precursor of a fresh eruption from its surface. Its diameter, perpendicular to the axis, was found to be less than 1600 miles.

There were three dark openings in the innermost envelope, between which it was intersected with bright rays. In Plate I., the engraver has given an eminently successful representation of the comet as it appeared in the field of the great refractor. The character of the light of the nebulosity composing the envelopes, and the appearance of the dark axial stripe penetrating, with well-defined outlines, quite up to the nucleus, have been preserved with great fidelity.

Fig. 12.

The long narrow ray first noticed on Sept. 25th, springing from the convex side of the tail, was seen on the 2d of October as represented in Fig. 12.

The dimensions of the envelopes were as follows: —

$n\,b' = 13200$ miles, $\quad n\,c' = 7500$ miles,

$b\,b'' = 3300$ " $\quad c\,c'' = 18900$ "

The breadth of the brightest part of the tail, at a distance of 144000 miles from the nucleus, was 90000 miles, and its extreme length 25° to 30°.

The next date of observation was the 4th. Another envelope

was then rising, having already attained a diameter of above nine thousand miles within forty-eight hours. Its peculiar form and the position of a dark spot, *S*, are given in Fig. 13. A dark space now separated the envelope $c\,c'\,c''$ from the new one. In Fig. 13, we have from the micrometer measurements,

Fig. 13.

$n\,c' = 8900$ miles,
$n\,d' = 3050$ "
$n\,S = 1800$ "
$c\,c'' = 23500$ "
$d\,d' = 9300$ "

The nucleus was smaller and less bright than on the 2d. The secondary tail was 35°, or thirty-four millions of miles long. On the 5th of October, the comet attained its greatest brilliancy. Its head was close to Arcturus, a star of the first magnitude, to which it was but little inferior in brightness, although the contrast in the *intensity* of their light was very evident. In Europe the two must have been seen still nearer to each other than they were in America, the nucleus passing a little to the south of the star, and the brightest part of the tail over it. The extremity of the train reached over Benetnasch and Mizar, the two southernmost stars in the tail of the Great Bear. It could be traced through an arc of 35°. Its breadth was 5° or 6°. With a little attention two additional streamers could be seen, one of

Fig. 14.

which was between 50° and 60° long, or above fifty millions of miles, with a slight curvature as in Fig. 14.

The interest of the telescopic view, taking all the circumstances into account, the size of the instrument, the perfect purity of the atmosphere, and the splendor of the object, have rarely been surpassed. The nucleus and the outline of its nearest envelope were visible in full sunshine with the large telescope. The head of the comet could be seen with the naked eye at twenty minutes after sunset, at which time the second envelope was discernible with the telescope. It is most remarkable, that, with all this accession of brightness, the nucleus itself had now diminished to a diameter of only four or five hundred miles, scarcely one fifth of what it was on the morning of the 9th of September, by a very careful determination. Its volume had thus diminished to *one twentieth* part only. The remaining nineteen twentieths had, in the intervening period, expanded into the tail, or had gone to form the envelopes which now encircled it, by a process which has been fully illustrated in the preceding pages. But are we then to conclude that the nucleus, the focus of these mysterious operations, had in this way expended the greater part of its substance? To this inquiry the best reply is a consideration of its subsequent condition. After several more eruptions from its surface, similar to those above described, it receded from our view about the 20th of October, with an evident *increase* of size compared with its condition two weeks before, and still shining with its accustomed intensity.

Examined in the daytime on the 5th with the highest powers which it would bear, no indication of a *phase* could be seen. The dark spot at *S*, of Fig. 13, had expanded in about the same proportion with the whole envelope in which it was situated. From near the vertex, and from the sides of the latter, there seemed to be an escape of jets of luminous gas, which streamed off like light spray thrown up against an opposing wind and driven before it.

The dimensions of the envelopes were as follows: —

$n d' = 4200$ miles, $\quad n c' = 9500$ miles,

$d d'' = 8900$ " $\quad c c'' = 27000$ "

$n b' = 14200$ miles,

$b b'' = 40500$ "

The diameter $d d''$ passed considerably above the nucleus. The outline $a a' a''$ of previous figures has faded away, and it is uncertain whether $b b' b''$ is the margin of the envelope so designated, or only that of the pretty sudden terminus of the stronger light comprising $a a' a''$ as well. The extreme diffusion of the light at some of the points measured, occasions a good deal of uncertainty in the above numbers; the distances to the vertices are usually the most trustworthy.

It will not be necessary to enter into the details of the history of other envelopes further than to indicate some of their leading features. Between the 2d and the 20th of October inclusive, four of them rose in succession from the nucleus. One, which was first seen on the 4th as just described, one between the 8th and 9th, another on the 15th, and a fourth on the 20th. The outlines of the brighter parts of three of them are shown in figures 15, 16, 17, 18. Plate II. has been engraved from a drawing of the comet selected from a series

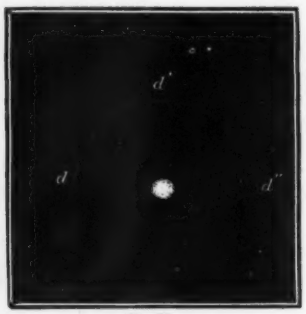

Fig. 15.

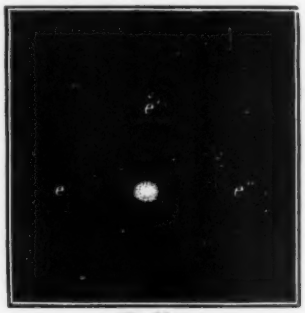

Fig. 16.

executed with great care by Mr. Henry G. Fette. It gives a view of the region surrounding the nucleus on the evening of Oct. 10, when

Comet of Donati Oct.r 4th 1858

it was at its least distance from the earth, and as it appeared in the field of the great refractor of the Observatory of Harvard College.

Fig. 15 is the outline on Oct. 6th of the envelope which made its appearance between the 2d and 4th; the places are indicated where it was intersected by brighter rays; its interior structure was very irregular. In Fig. 16, we have a representation of the inner envelope on Oct. 11th, the brighter portions only being included. Fig. 17 gives the outlines of the envelope first seen on Oct. 15th. Fig. 18 shows it at a more advanced stage, three days later.

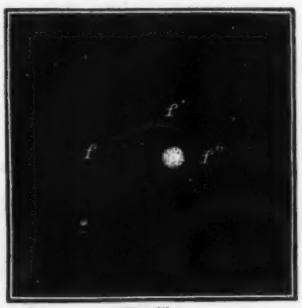

Fig. 17.

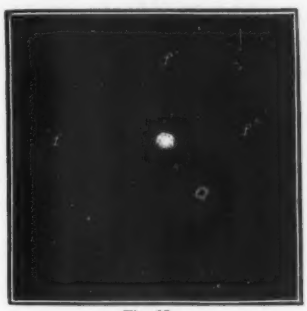

Fig. 18.

A change, in the relative proportion of light distributed on the two sides of the principal axis of the comet, had been progressing up to about the 6th of October. At this date, although there may still have been a little more light on the right hand side, the difference was not nearly so large as it had been. The diameter of the nucleus was then 800 miles. On the 8th, its diameter was 1100 miles. The envelopes were most distinct on the left hand, or preceding side. The change was a permanent one, and for the future this became the brightest half of the head of the comet. It is curious to observe a corresponding change in the inclination of the envelopes to the axis. They now inclined even more decidedly to the left hand than they had at first done to the opposite side. The two last, those of Oct. 15th and 20th, seem in fact in the first instance to have issued as luminous jets or streams from the side, rather than from the

vertex of the nucleus. A similar reversal in the order of brightness was evident in the part of the train near the nucleus.

The tenth was the day of nearest approach to the earth, but the comet was manifestly on the wane, though expanded over a larger extent of the sky than before. Five envelopes, reckoning the exterior haze as one, could be traced through the whole or some part of their outline. The dark stripe of the axis was becoming less conspicuous, the central regions of the train being occupied with diffused light; on the 11th it was barely discernible. The last of the envelopes was thrown off on the 20th. The comet had now passed far to the south, and its low altitude prevented the continuance of the observations.

We must add a few words on the appearance presented by the tail between the 6th and the 10th of October. At the date first named, one of the supplementary rays, Fig. 19, attained a distance of 55°, or fifty millions of miles from the nucleus, somewhat exceeding that of the principal tail, and in a direction, as usual, nearly in a line from the sun. Others less perfectly developed could be discerned near a point where the curvature of the main stream was pretty suddenly changed. On the 8th, Fig. 20, five or six transverse bands could be distinguished in the tail half a degree or less in breadth, with clear, well-defined outlines, and perfectly resembling auroral streamers, excepting that they kept their position permanently, that is, without motion

Fig. 19.

sensible to the eye, they diverged from a point between the sun and the nucleus. The supplementary ray was not inserted in the original drawing from which Fig. 20 was engraved. Its place in the cut has been supplied from sketches on the 9th, and dates previous, allowing for its motion in the interval.

Fig. 20.

The train attained its largest apparent dimensions on the 10th, when the main stream of light could be distinguished through an arc of 60°, corresponding to a length of fifty-one millions of miles, or rather more than half the distance of our earth from the sun. The distribution of its light at a distance of 20° or 30° from the nucleus in parallel or slightly diverging bands, alternating with dark spaces, was strongly exhibited. They were 5° long, and 20′ or 30′ wide, and might aptly be compared either to the streamers which often break up the continuity of an auroral arch, or to a collection of five or six tails of small comets, forming from the remains of the large one. Whatever may have been their real nature, the impression to the eye involuntarily suggested the comparison. These bands were visible for one or two succeeding evenings, but were soon overpowered by the moonlight.

We will conclude with a review of some particulars relating to the comet which seem to deserve special attention. The dimensions

of the tail, and of the nucleus and envelopes on the several dates of observation, are given below. Apparent variations in the size of the nucleus were sometimes caused by disturbances in our own atmosphere, but in most cases the changes were undoubtedly real ones. The presence of moonlight, or of the slightest haze in the sky, had a very perceptible effect in diminishing the arc through which the tail could be traced. This will sufficiently explain the irregularities noticed in comparing its proportions from night to night.

Date.	Length of tail	Breadth at extremity.	Remarks.
1858. Aug. 29	2° = 14,000,000 miles.		
" Sept. 8th and 9th	4 = 16,000,000 "		
" " 12	6 = 19,000,000 "		
" " 17	4 = 10,000,000 "		Moonlight.
" " 23	7 = 12,000,000 "		"
" " 24	7 = 12,000,000 "		"
" " 25	11 = 17,000,000 "	1,500,000 miles.	
" " 27	13 = 18,000,000 "		
" " 28	19 = 26,000,000 "		
" " 30	22 = 26,000,000 "	3,000,000 "	
" Oct. 2	25 = 27,000,000 "	5,000,000 "	
" " 5	35 = 33,000,000 "	5,000,000 "	
" " 6	50 = 45,000,000 "		
" " 8	50 = 43,000,000 "	7,000,000 "	
" " 10	60 = 51,000,000 "	10,000,000 "	
" " 12	45 = 39,000,000 "		
" " 15	15 = 14,000,000 "		Moonlight.

Date.	Length of 'Streamers.'	Breadth at extremity.
1858. Oct. 4	35° = 34,000,000 miles.	1,000,000 miles.
" " 5	55 = 53,000,000 "	" "
" " 6	55 = 50,000,000 "	" "

In computing the above, the curvature has not been regarded.

It must be borne in mind that we have taken for the extremity of the tail, the furthest point at which it was possible to detect a trace of it. It would scarcely have been noticed beyond 30° or 35°, even between the 5th and 10th of October, without a particular effort of the attention, and some training of the eye. The streamers, or additional rays, might easily have escaped notice altogether from their faintness. In making a comparison of the size of this comet with others, it will be best to limit the extent of the tail to the arc over which it was plainly visible, which would give a length of about thirty-five millions of miles. The shortening of the tail between the 12th and the 17th of September, is due entirely to the effect of moonlight. The more abrupt change between the 10th and 15th of October is partly due to the same cause, but there must also have been a great diminution in brilliancy.

For the nucleus we have the following measured diameters: —

1858, July 19. Diameter $5'' = 5600$ miles. This probably includes the dense nebulosity immediately surrounding it, not distinguishable at the time from the true centre, on account of the low altitude of the comet.

Aug. 19. "Nucleus equals a star of the seventh magnitude."

" 29. Head of the comet visible to the unassisted eye as a star of the sixth magnitude.

Aug. 30. Diameter $6'' = 4660$ miles. This result perhaps includes more than the true nucleus.

Sept. 8, 9. Diameter $3'' = 1980$ miles. Taken just before sunrise, when all of the comet, excepting the nebulosity next outside, which was 3300 miles in diameter, was obliterated. On a dark sky the apparent diameter was 5280 miles, and the light equivalent to that of a star of the fifth magnitude.

Sept. 12. To the naked eye the head of the comet appeared as a star of the third magnitude.

On Sept. 17th, it equalled a star of the second magnitude.

Sept. 23. "To the naked eye the head of the comet is brighter than a star of the first magnitude." Its brilliancy at this date (one week before its perihelion passage, and seventeen days before its nearest approach to the earth), had reached a maximum. It is interesting to remark, that between the 17th and 23d was first noticed the characteristic formation of envelopes, which plainly operated as a check upon the accumulation of brightness at the central point. The nucleus, during the remaining period of its visibility, went through a series of periodic changes, acquiring more light just before an eruption, and suddenly diminishing after it. The variations, although evident to the eye, could not be accurately measured on account of the smallness of the angle subtended, and its want of precise definition.

On the 23d, its diameter, which appeared to be less than usual, was $3'' = 1280$ miles.

Sept. 24. Diameter $2''.5 = 1030$ miles.

Oct. 2. Diameter $5''.2 = 1560$ "

" 4. Nucleus evidently smaller than on the 2d.

" 5. Diameter $1''.5 = 400$ miles; "it is certainly less than $2'' = 540$ miles." This determination was made under most favorable conditions.

Oct. 6. Diameter $3'' = 800$ miles. The head of the comet nearly equalled Arcturus.

Oct. 8. Diameter $4''.4 = 1120$ miles. "The nucleus is decidedly brighter than on the 6th, and is preparing to throw off a new envelope."

Oct. 9. The nucleus had diminished in size simultaneously with the appearance of a new envelope.

Oct. 10. Diameter 2″.5 = 630 miles.

" 11. Diameter 2″ = 510 "

" 15. The head of the comet was as bright to the naked eye as a star of the third magnitude.

Oct. 18. Diameter 3″ = 900 miles.

" 19. Diameter 3 = 920 miles. The nucleus was compared with three stars of the sixth magnitude at the same altitude, and found to be far brighter than either of them. It was probably at least as bright as a star of the fifth magnitude, while to the naked eye the head nearly equalled one of the third magnitude.

Oct. 20. Diameter 2″ = 660 miles. A new envelope was forming.

As before remarked, the least observed diameter of the nucleus, 400 miles, occurred on October 5th, the evening when the comet reached its maximum of brightness.

In order to exhibit the progressive motion of the envelopes from their point of origin, we give below in one view the distances of their vertices from the nucleus at different dates. The distances were measured in the line from the nucleus towards the sun. The better to distinguish them, we will use the following notation, which is the same with that employed in the wood-cuts.

a' =	Vertex of envelope	first	seen	on	Sept. 20.
b' =	"	"	"	"	23.
c' =	"	"	"	"	27.
d' =	"	"	"	Oct.	4.
e' =	"	"	"	"	9.
f' =	"	"	"	"	15.
g' =	"	"	"	"	20.

Distances in miles from the nucleus (n) to the vertices of the envelopes a, b, and c: —

	na'	nb'	nc'	nd'	ne'	nf'
Sept. 23	*13000	*6400				
" 24	13400	5800				
" 25	*18000	7100				
" 27		8400	3500			
" 29		10500	6000			
Oct. 2		13200	7500			
" 4			8900	3050		
" 5		14200	9550	4210		
" 6			10100	4270		
" 8			12400	7160		
" 9			13200	8650	1910	
" 10			14100	8780	2760	
" 11				*10200	*4200	
" 15				*11400	8160	3200
" 18				*14500	9950	4400
" 19					11200	5500

The vertex g' had barely left the surface of the nucleus on the 20th.

The comet of DONATI, although surpassed by many others in size, has not often been equalled in the intensity of the light of the nucleus. The diameter of the surrounding nebulosity on the other hand was unusually small, never much exceeding one hundred thousand miles, while that of the great comet of 1811, was ten times larger — its envelope attaining an elevation of more than three hundred thousand miles above the central body, exceeding by more than twenty times the largest of our measurements given above. Still it would be difficult to instance any one of its predecessors which has combined so many attractive features.

Its early discovery enabled astronomers, while it was yet scarcely distinguishable even with the telescope, to predict, some months

---

* The numbers marked with an asterisk are less reliable than the others.

in advance, the more prominent particulars of its approaching apparition, which was thus observed with all the advantage of previous preparation and anticipation. The perihelion passage occurred at a most favorable moment for presenting the comet to good advantage. When nearest the earth, the direction of the tail was nearly perpendicular to the line of vision, as represented in Fig. 21, so that its proportions were seen without foreshortening. The position of the comet relatively to the sun and to the earth at the date of discovery, June 2d, and when nearest the earth, Oct. 10th, will be understood from the figure. Its situation in the latter part of its course afforded also a fair sight of the curvature of the train, which seems to have been exhibited with unusual distinctness, contributing greatly to the impressive effect of a full length view. Frequent allusion has been made to the influence of the light of the moon on the visibility of the comet. Few readers will be aware how much of its splendor and vast dimensions, during the first ten days of October, we owe to the fortunate circumstance that, at this critical period, the moon was absent from our evening skies. The effect of the presence of a full moon, though simply optical, and due only to the force of contrast, would have been quite as prejudicial as if the comet had lost two thirds of its train, and as large a proportion of the brightness of the remaining third; above all, we must have lost those most singular phenomena — the supplementary rays, and the alternating bright and dark bands in the train; the latter seem to have been new in cometary history. Supposing the substance of the tail to be driven off into space, never again to return to its orig-

Fig. 21.

inal source, the inquiry at once arises, What then becomes of it? The appearances in question show plainly enough a process of separation into distinct masses, and in each of these a tendency to condense about a central axis.

It is remarkable that the aggregation should have been around separate axes, rather than about one or more central points, and that the axes should have manifested a disposition to diverge from the sun; as though these collections of nebulosity were in reality a group of new comets in process of formation. The increasing moonlight and low altitude of the comet would not allow their being followed to a more complete development.

The condition of the nucleus and neighboring region has received a large share of attention in the preceding pages, because it has afforded so ample an illustration of phenomena of which, up to the present time, very little has been certainly known. The comets of 1744 and of 1811 had well-formed envelopes, but the observations upon them were too imperfect and disconnected to afford much more than a basis for conjecture as to their origin and destination. That of Halley, at its apparition in 1835–36, furnishes an example more nearly parallel to the present one, but its phenomena were on a comparatively feeble scale.

The most recent intelligence leaves no room to doubt that the comet of Donati is periodical, having a time of revolution of about two thousand years. The following are the results arrived at by different computers: —

Watson,	2415	years.
Bruhns,	2102	"
Löwy,	2495	"
Graham,	1620	"
Brünnow,	2470	"
Newcomb,	1854	"

The last two determinations are based upon longer intervals of observation than the others, Mr. Newcomb's being a few days longer than that of Dr. Brünnow. The remaining uncertainty in the period will be materially reduced, when observations have been received from the southern hemisphere, where the comet is still in sight.

The subjoined table contains the distances of the comet from the sun, and from the earth, and its hourly rate of motion: —

1858.	Distance from Sun in miles.	Distance from Earth in miles.	Hourly Velocity in miles.
June 2,	215,000,000	240,000,000	65,000
July 2,	173,000,000	240,000,000	72,000
Aug. 2,	127,000,000	220,000,000	84,000
Sept. 1,	82,000,000	160,000,000	105,000
" 11,	70,000,000	130,000,000	115,000
" 21,	60,000,000	95,000,000	124,000
Oct. 1,	56,000,000	66,000,000	128,000
" 11,	61,000,000	52,000,000	123,000
" 21,	71.000,000	67,000,000	114,000

Supposing its last perihelion passage to have occurred at the beginning of the Christian era, it must have passed its aphelion in the early part of the tenth century, at a distance of 14300 millions of miles from the sun, its velocity at that point being 480 miles an hour.

Mathematical Monthly Notices.

An Elementary Treatise on Mechanics; designed as a text-book for the University Examinations for the ordinary degree of B. A. Part I. Statics. Part II. Dynamics of a Particle. By J. B. CHERRIMAN, *M. A., late Fellow of St. John's College, Cambridge, and Professor of Natural Philosophy in University College, Toronto.* MACLEAR & Co., King street East. 1858.

THE author says: "As my design has been only to furnish to students a text-book for such parts of the subject as are required for the ordinary degree of B. A. in Universities, I have not thought it advisable to burden this work with mere explanation or illustration, or to add examples; presuming that such, where necessary, will be furnished in the lecture-room by the tutor."

We have read this work with sufficient care to be satisfied that it meets a want in the usual undergraduate course of instruction. Comparatively few students acquire sufficient knowledge of the higher mathematics, to be able to read with much advantage a work on mechanics, in which the notation and logic of the calculus is used; while the elementary mathematics should be far too thoroughly learned by all, not to fit the student to read something more rigorous on the subject than is found in our ordinary works on Natural Philosophy. We have here as full a work on Mechanics as the student will be likely to have time to accomplish during his college course, which demands a knowledge of only the simplest elements of Mathematics. Indeed, in many of our Academies and High Schools, the mathematical standard is sufficiently high to enable students to read this work without much difficulty, and with far greater profit than they usually derive from a merely descriptive or experimental course of Mechanics.

The work is well arranged, the definitions and demonstrations are clear and concise, and our only regret is, that the author did not think best to collect under each head a few carefully chosen examples.

Besides the fact, that every teacher who supplies this deficiency performs the labor which the author might have saved to all, we are afraid that far too many will want the time or disposition to supply it at all, and thus the student will fail to reap all the advantage he ought to derive from the study of the work.

Journal de Mathématiques Pures et Appliquées, ou Recueil Mensuel de Mémoires sur les diverses parties des Mathématiques; Publié Par JOSEPH LIOUVILLE, *Membre de l'Académie des Sciences et du Bureau des Longitudes, Professeur au Collége de France.* Paris: MALLET-BACHELIER, Quai des Augustines, No. 55.

For those who have been in the habit of consulting the pages of this valuable Journal during the twenty-three years of its existence, this brief notice is unnecessary; and we only insert it, for the information of the younger portion of our readers, as part of a plan in which we propose to give brief historical notices of all the Mathematical Journals at present existing, as well as of those which have been discontinued. Besides those which have existed in our own country, there are quite a number of discontinued journals in Europe of far too great value not to be generally known, and especially by those who have any interest in tracing the progress of the Mathematics since the days of Newton and Leibnitz.

The "Annales" of M. Gergonne, to which we shall hereafter refer more particularly, was discontinued in 1831, after a most successful issue of twenty years. In the "Avertissement" M. Liouville announces his Journal as the continuation of the "Annales," and adds, "Our Journal, like that of M. Gergonne, will be a monthly. The first number will appear in January, 1836, and the following ones from month to month, with all desirable punctuality. The numbers will be of unequal size, and vary from thirty-two to forty pages 4to, according to the nature of the memoirs which they contain. Together they will form a large volume each year, containing all the plates necessary for the understanding of the text."

How completely this prediction of the Editor has been fulfilled, the twenty-three volumes, completed with the issue of the number for December, 1858, averaging over 450 quarto pages per volume, sufficiently attest.

In the "Avertissement" to the twenty-first volume (1856), M. Liouville remarks: "I established this Journal at a time when the facilities of publication for young geometers were much less than at present. We had, it is true, Crelle's Journal and that of the Polytechnic School; but the *Correspondence* of M. Hachette, the *Annales* of M. Gergonne, the *Bulletin des Sciences* of M. Férussac, &c., had been discontinued for many years. Our first number was for January, 1836; and it was only six months after, at its second semester, that the Academy of Sciences conceived its *Comptes Rendes*, afterwards imitated by the majority of the Learned Societies. We have, since that time, issued a volume each year; the first twenty volumes composing the first series. The one we now offer to the public begins a new series."

The number for December, 1858, will complete the third volume of the new series. It will give only an indefinite idea of the contents of this Journal to add, that its first twenty-two volumes contain eight hundred Notes and Memoirs, in over ten thousand large quarto pages, comprising contributions from nearly all the eminent European mathematicians of the last quarter of a century.

Editorial Items.

THOSE wishing to secure the services of a gentleman of large experience, eminently successful both as an author and teacher, and for many years Professor of Mathematics in one of the oldest and most influential Colleges in the country, will please apply to the Editor of the Mathematical Monthly for further information. . . . . ASAPH HALL, Esq., Assistant at Harvard College Observatory, sends the following erratum: — Article 32, page 35 of DAVIS's Translation of GAUSS's "Theoria Motus," for $\tan \frac{1}{2} v = \frac{(u-1)\tan \frac{1}{2}\psi}{u+1}$ read $\tan \frac{1}{2} v = \frac{u-1}{(u+1)\tan \frac{1}{2}\psi}$. This error is also in the original. . . . . In reply to numerous inquiries concerning publications of interest to mathematicians, issued in this country, we subjoin the following NAUTICAL ALMANAC PUBLICATIONS: Tables of the Moon, constructed from PLANA's Theory, with AIRY's and LONGSTRETH's corrections; HANSEN's two inequalities of long period arising from the action of Venus, and HANSEN's values of the secular variations of the mean motion, and of the motion of the perigee. Arranged for the use of the Nautical Almanac in a form designed by Professor BENJAMIN PEIRCE, Consulting Astronomer, under

the superintendence of CHARLES HENRY DAVIS, Lieutenant United States Navy; and published under the authority of the Hon. JOHN P. KENNEDY, Secretary of the Navy. Washington: 1853. Nautical Almanac and American Ephemeris for the years 1855 to 1860, inclusive; the volume for 1861 being nearly ready. The nautical part, for the years 1855 to 1861, inclusive, has been published separately, especially for the use of navigators. The sale of this edition has reached six thousand copies per annum. The volume of the American Ephemeris for 1857 contains Prof. CHAUVENET'S valuable methods for "Correcting Lunar Distances," and for "Finding the Error and Rate of a Chronometer by Equal Altitudes;" also "Logarithms of the Le Verrier Coefficients of the Perturbative Function of Planetary Motion." By the late SEARS C. WALKER..... In future numbers of the Monthly we shall give the titles of all Memoirs upon mathematical subjects ever published in this country. If authors, publishers, or societies publishing Memoirs, will in future send us the titles, in full, of works or memoirs in any departments of the Pure or Applied Mathematics, as soon as published, we will insert them in the Monthly..... It gives us pleasure to add the following names to our list of coöperators and contributors. M. L. COMSTOCK, Esq., Tutor in Knox College, Galesburg, Ill.; W. W. DICKSON, Esq., Pittsburg, Pa.; Prof. MARK D. HANOVER, Principal of Liberty Academy, Springfield, Tenn.; R. G. HATFIELD, Esq., Architect, N. Y.; EPHRAIM HUNT, Esq., Sub-master English High School, Boston; Prof. LEM. JOHNSON, Normal College, N. C.; C. J. KEMPER, Esq., Harrisburg, Va.; Prof. N. H. LOSEY, Knox College, Galesburg, Ill.; Prof. SAMUEL SCHOOLER, Edge Hill Academy, Guiney's P. O., Va.; JOHN B. THOMPSON, Esq., Charlottsville, Va.; Rev. A. D. WHEELER, Brunswick, Maine; G. T. Kingston, M. A., Director of the Magnetic Observatory, Toronto, Canada; Prof. WRAY BEATTIE, Iowa Wesleyan University, Mount Pleasant; Prof. J. T. BENEDICT, New York Free Academy; H. WILLEY, Esq., New Bedford, Mass.; JOHN W. BULKLEY, Esq., Superintendent of Schools, Brooklyn, New York..... BOOKS RECEIVED. Nouvelles Annales de Mathématiques for October, 1858. Paris: Mallet-Bachelier. Elements of Descriptive Geometry. Part I. The Point, the Straight Line, and the Plane. By SAMUEL SCHOOLER, M. A. J. W. Randolph, 121 Main street, Richmond, Va., 1853. We shall be much obliged if every institution of learning in the country, publishing a catalogue, will send a copy to the Mathematical Monthly..... We trust our friends will pardon any want of punctuality on our part, in answering letters, or acknowledging the receipt of communications. We should be happy to answer at once, in all cases; but this, with our other duties, is simply impossible; and, therefore, as a rule, we only reply in those cases, in which there seems to be an immediate necessity, and the reception of the following number of the Monthly will not be a sufficient reply..... The terms of the Monthly are strictly and invariably in advance, for the simple reason that it is absolutely necessary for us to know upon what we are to rely to meet the year's expenses; and we hope, therefore, to be able to send receipted bills in the January Number to those whose subscription now remains unpaid.

December, 1858.

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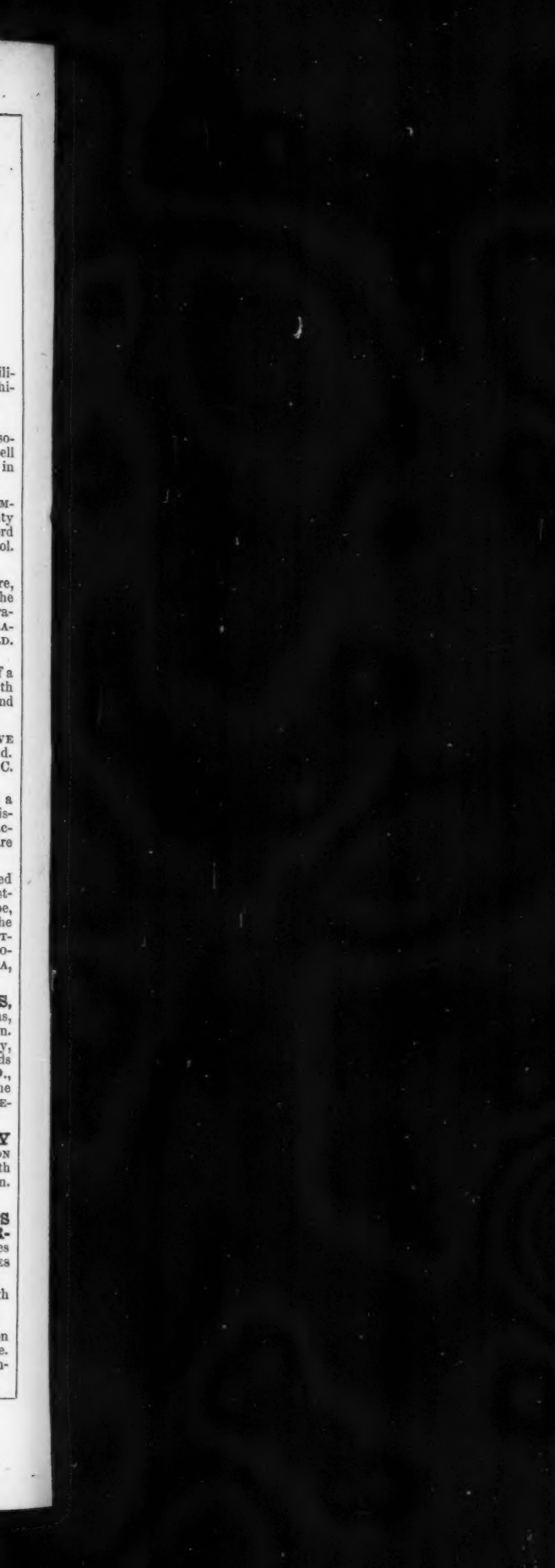

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